ΕΝΔΕΙΚΤΙΚΕΣ ΑΠΑΝΤΗΣΕΙΣ ΣΤΗ ΦΥΣΙΚΗ ΠΡΟΣΑΝΑΤΟΛΙΣΜΟΥ 12 06 2019

ΘΕΜΑ Α

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ΘΕΜΑ Β

Β1. $f_1 = \dfrac{u_{nx}}{u_{nx} + \dfrac{u_{nx}}{20}} f_s = \dfrac{u_{nx}}{\dfrac{21}{20} u_{nx}} f_s$

$$\Leftrightarrow f_1 = \frac{20u_{nx}}{21u_{nx}} f_s \Leftrightarrow f_1 = \frac{20}{21} f_s \,.$$

ΑΔΟ για την πλαστική κρούση: $m_1 u_s = \left(m_1 + m_2\right) V \Leftrightarrow V = \dfrac{u_S}{2} \Leftrightarrow V = \dfrac{u_{nx}}{40}$

$$f_2 = \frac{u_{nx}}{u_{nx} + \dfrac{u_{nx}}{40}} f_s \Leftrightarrow f_2 = \frac{40}{41} f_s$$

$$\frac{f_1}{f_2} = \frac{\dfrac{20}{21} f_s}{\dfrac{40}{41} f_s} = \frac{41}{42}$$

Σωστό το ii.

B2. Εφ' όσον έχει σταθεροποιηθεί η στάθμη του δοχείου, η παροχή από το Γ (Π_2) θα ισούται με την παροχή από το Ζ (Π_3)

Άρα $A_2 u_2 = A_3 u_3$ (1)

Όμως από εξίσωση συνέχειας $A_1 \cdot u_1 = A_2 \cdot u_2 \Rightarrow 2A_2 u_1 = A_2 u_2 \Rightarrow u_2 = 2u_1$ (2)

Από Bernoulli από Δ → Γ : $p_\Delta + 0 + \frac{1}{2}\rho u_1^2 = p_{\alpha\tau\mu} + \frac{1}{2}p u_2^2$ (3)

Όμως $p_\Delta = p_{\alpha\tau\mu} + pgh$ (4)

(3) $\overset{(4)}{\underset{(2)}{\Rightarrow}} p_{\alpha\tau\mu} pgh + \frac{1}{2}p \cdot \frac{u_2^2}{4} = p_{\alpha\tau\mu} + \frac{1}{2}pu_2^2 \Rightarrow \not{\rho} gh = \frac{3}{8}\not{\rho} u_2^2 \Rightarrow (\not{5})\ u_2^2 = \frac{8g}{3}(5)$

Στο δοχείο από Torricelli $u_3 = \sqrt{2gH}$ $u_3^2 = 2gH$ (6)

$(1) \overset{(5)}{\underset{(6)}{\Rightarrow}} \not{A}_2 \cdot \sqrt{\frac{8\not{g}h}{3}} = \frac{A_2}{2} \cdot \sqrt{2gH} \Rightarrow \sqrt{\frac{8h}{3}} = \sqrt{\frac{H}{2}} \Rightarrow \frac{h}{H} = \frac{3}{16}$

Σωστό το iii.

B3. ΘΜΚΕ Α → Δ: $K_\Delta - K_A = W_F \Rightarrow \frac{1}{2}I\omega^2 - 0 = F \cdot l \cdot \Delta\varphi \Rightarrow$

$\Rightarrow \frac{1}{\not{2}} \cdot \frac{Ml^2}{3} \cdot \omega^2 = F \cdot l \frac{\pi}{2} \Rightarrow \omega^2 = \frac{3\pi F}{Ml} \Rightarrow \omega = \sqrt{\frac{\not{3}\pi \cdot 9\pi}{\not{3}\pi \cdot 1}} = 3\pi$ rad/s

Κατά την κρούση επειδή $\Sigma\tau_{\varepsilon\xi} = \vec{0}$ ισχύει:

Lαρχ=Lτελ. $\Rightarrow I\omega = I_{\sigma\upsilon\sigma} \cdot \omega_1 \Rightarrow \frac{M\ell}{3} \ldots \left(\frac{\ell}{3} \ldots \ell \right)$

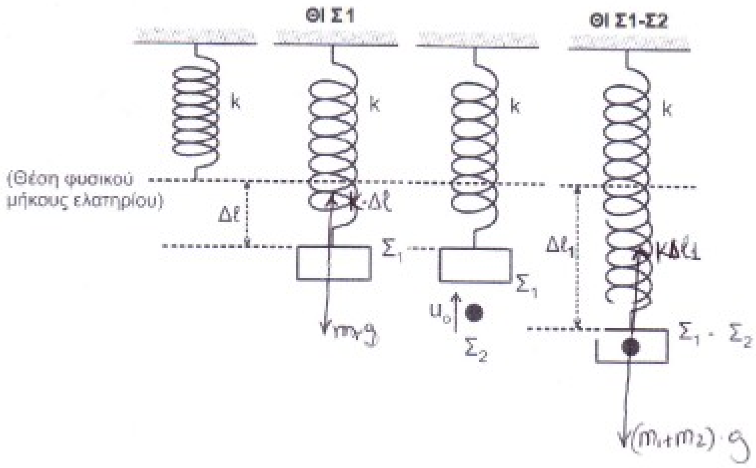
$\Rightarrow \omega_1 = \frac{M}{M + 3m} \cdot \omega \Rightarrow$

$\Rightarrow \omega_1 = \frac{\omega}{2} \Rightarrow \omega_1 = 3\frac{\pi}{2}$ rad/s

$\Delta\theta = w\Delta t \Rightarrow \Delta t = \frac{\Delta\theta}{w} = \frac{\frac{\pi}{2}}{\frac{3\pi}{2}} = \frac{1}{3}s$

Σωστό το ii.

ΘΕΜΑ Γ



Γ1. $\Theta I_1 : \Sigma F = 0 \Rightarrow \mathrm{K}\Delta\ell$ (1)

$\Theta I_2 : \Sigma F = 0 \Rightarrow \mathrm{K}\Delta\ell \quad m_2)\cdot g \Rightarrow \mathrm{K}\Delta\ell$ $\quad \Rightarrow \Delta\ell \quad \ell$ (2)

$(1): \mathrm{K} = \frac{mg}{\Delta\ell} \quad 10 \quad = 200\, N/m$

Επειδή το συσσωμ. σταματά στη θέση ΦΜ, αυτή είναι ακραία

$\mathrm{A} = \Delta\ell \quad \ell$

Γ2. ΑΔΕ: Υταλ. + $\mathrm{K} = E_{ολ} \Rightarrow \frac{1}{2}Dy^2 + \frac{1}{2}2m_1u^2 = \frac{1}{2}DA^2$ (3)

Αμέσως μετά την κρούση το συσσωμάτωμα έχει απομάκρυνση $y = \Delta\ell \quad \ell$ και ταχύτητα u.

$(3) \Rightarrow D\left(A^2 - y^2\right) = 2m_1 \cdot u^2 \Rightarrow u = \sqrt{\frac{K}{2m_1}\left(A^2 - y^2\right)} \Rightarrow u = \frac{\sqrt{3}}{2} m/s$

ΑΔΟ: $m_2u_0 = \left(m_1 + m_2\right)u \Rightarrow u_0 = 2_u \Rightarrow u_0 = \sqrt{3}\, m/s$

$\mathrm{K}_2 = \frac{1}{2}m_2u_0^2 \Rightarrow \mathrm{K}_2 = \left(\frac{1}{2}\cdot 1\cdot 3\right)J \Rightarrow \mathrm{K}_2 = 1,5J$

Γ3. $\Delta\vec{\rho}_2 = \vec{\rho}_{2,e} - \vec{\rho}_{2,a} \Rightarrow \Delta\vec{\rho}_2 = m_2\vec{u} - m_2\vec{u}_0 \Rightarrow$

$$\Delta\rho_2 = m_2\frac{u_0}{2} - m_2u_0 \Rightarrow \Delta\rho_2 = -\frac{m_2u_0}{2} \Rightarrow$$

$$\Rightarrow \Delta\rho_2 = -\frac{1\cdot\sqrt{3}}{2} \Rightarrow \Delta\rho_2 = -\frac{\sqrt{3}}{2}\ \text{kgm/s}$$

Άρα $\left|\Delta\vec{\rho}_2\right| = \frac{\sqrt{3}}{2}$ kgm/s με κατεύθυνση αρνητική δηλαδή προς τα κάτω.

Γ4. Αμέσως μετά y = +0,05m με $u = \frac{\sqrt{3}}{2} m/s > 0$

Επίσης $D = \mathrm{K} = (m_1 + m_2)\omega^2 \Rightarrow \omega = \sqrt{\frac{\mathrm{K}}{m_1 + m_2}} \Rightarrow \omega = 10\,\text{rad/s}$

$$\psi = \mathrm{A}\eta\mu(\omega t + \phi_0) \overset{t=0}{\Rightarrow} 0{,}05 = 0{,}1\eta\mu\phi_0$$

$$\overset{0\le\phi_0\angle 2\pi}{\Rightarrow} \varphi_0 \begin{cases} \pi/6 \\ \text{ή} \\ 5\pi/6 \end{cases}$$

όμως u>0 $\Rightarrow u_{\max}\sigma\upsilon\nu\varphi_0 > 0 \Rightarrow \sigma\upsilon\nu\varphi_0 > 0 \Rightarrow \phi_0 = \frac{\pi}{6}$

άρα $y = 0{,}1\eta\mu\left(10t + \frac{\pi}{6}\right)$ (s.I)

$$(4) + (5) \rightarrow 2\mathrm{T}4 - \mathrm{M}\kappa g\eta\mu\phi = \frac{3}{2}\mathrm{M}\kappa\alpha\alpha \Rightarrow$$

$$\Rightarrow \mathrm{T}4 = \frac{\mathrm{M}\kappa g\eta\mu\phi}{2} + \frac{3}{4}\mathrm{M}\kappa\alpha\alpha(6)$$

$$\mathrm{T}\rho:\ \Sigma\tau = \mathrm{I}_{\tau\rho}\alpha\gamma\omega\nu_{\tau\rho} \Rightarrow \mathrm{T}3R\tau - \mathrm{T}4R\tau = \frac{\mathrm{M}\tau R\tau^2}{2}\alpha\gamma\omega\nu_{\tau}$$

$$\mathrm{T}3 - \mathrm{T}4 = \frac{\mathrm{M}\tau R\tau}{2}\alpha\gamma\omega\nu_{\tau}(7)$$

$$\Sigma : \Sigma F = \mathrm{M}_\Sigma\alpha_\Sigma \Rightarrow \mathrm{M}_\Sigma g - \mathrm{T}3 = \mathrm{M}_\Sigma\alpha_\Sigma \Rightarrow$$

$$\Rightarrow \mathrm{T}3 = \mathrm{M}_\Sigma g - \mathrm{M}_\Sigma\alpha_\Sigma(8)$$

$$(7)\overset{(6)}{\underset{(8)}{}}\; \mathrm{M}_\Sigma g - \frac{\mathrm{M}\kappa g\eta\mu\varphi}{2} = \mathrm{M}_\Sigma \alpha_\Sigma + \frac{3}{4}\mathrm{M}_\kappa \alpha_\alpha + \frac{\mathrm{M}\tau R\tau}{2}\alpha_\alpha (9)$$

όμως $\left.\begin{cases} \alpha_\Sigma = \alpha_{\mathrm{E,Z}} = \alpha\gamma\omega\nu_\tau \cdot R\tau \\ \alpha_{\mathrm{Z,H}} = \alpha_\mathrm{N} \\ \alpha_\mathrm{N} = 2\alpha_\alpha \end{cases}\right\} \Rightarrow \alpha_\Sigma = \alpha\gamma\omega\nu_\tau \cdot R\tau = 2\alpha_\alpha (10)$

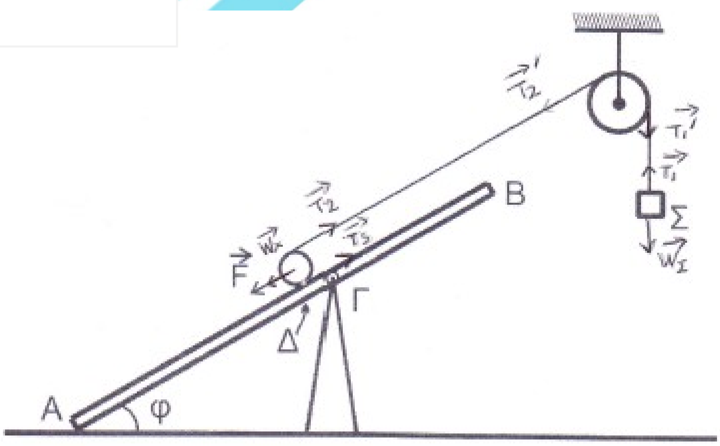
$$(9)\overset{(10)}{\Rightarrow}\left(\mathrm{M}_\Sigma - \frac{\mathrm{M}_\kappa \eta\mu\varphi}{2}\right)g = \left(\mathrm{M}_\Sigma + \frac{3}{8}\mathrm{M}_\kappa + \frac{\mathrm{M}_\tau}{2}\right)\alpha_\Sigma \Rightarrow$$

$$\Rightarrow \alpha_\Sigma = \frac{\left(2 - \frac{1}{2}\right)10}{2 + \frac{3}{4} + 1} = \frac{15}{\frac{15}{4}} m/s^2 = 4 m/s^2$$

$$(10) \Rightarrow \alpha_\alpha - \frac{\alpha_\Sigma}{2} = 2 m/s^2$$

<u>ΘΕΜΑ Δ</u>

Δ1.



$T_1 = T_1'$ και $T_2 = T_2'$ διότι τα νήματα είναι αβαρή και δεν ολισθαίνουν

Σώμα Σ: ΣF= 0 $\Rightarrow \mathrm{T}_1 = W_\Sigma \Rightarrow \mathrm{T}_1 = m_\Sigma \cdot g$ (1)

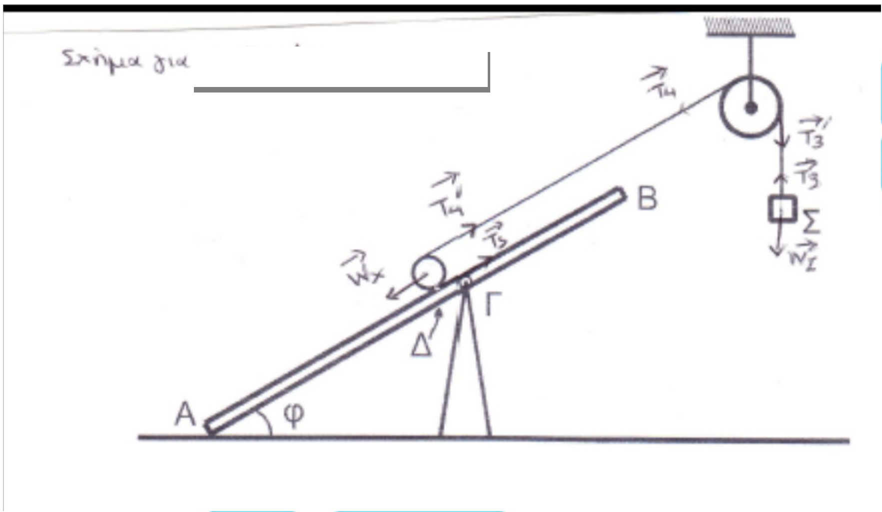
Τροχαλία: Στ=0 $\Rightarrow \mathrm{T}_1' \cdot R_\mathrm{T} = \mathrm{T}_2' R_\mathrm{T} \Rightarrow \mathrm{T}_1' = \mathrm{T}_2 \Rightarrow \mathrm{T}_1 = \mathrm{T}_2' \overset{1}{\Rightarrow} \mathrm{T}_2 = m_\Sigma \cdot g$ (2)

Κύλινδρος: Στ=0 $\Rightarrow$ $\mathrm{T}_2 \cdot R_T = T_S \cdot R_\kappa \Rightarrow \mathrm{T}_2 = \mathrm{T}_\Sigma$ (3)

$$\Sigma Fx = 0 \Rightarrow F + Wx - \mathrm{T}_2 - \mathrm{T}_s = 0 \underset{(|2)}{\overset{(3)}{\Rightarrow}} F + \mathrm{M}_\kappa g\eta\mu\varphi = 2m_\Sigma g$$

$$\Rightarrow F = (-\mathrm{M}_\kappa \eta\mu\varphi + 2m_\Sigma)g \Rightarrow F = 30N$$

Δ2.



Κύλινδρος: $\Sigma F_\mathrm{X} = \mathrm{M}_\kappa \alpha_{CM} \Rightarrow \mathrm{T}_4 + \mathrm{T}_S - \mathrm{M}_\mathrm{K} g\eta\mu\varphi = \mathrm{M}_\kappa \alpha_{CM}$ (4)

$$\Sigma\tau = \mathrm{I}\alpha_{\gamma\omega\nu} \Rightarrow \mathrm{T}_4 \cdot R_\kappa - \mathrm{T}_S R_\kappa = \frac{\mathrm{M}_\kappa R_\kappa^2}{2}\alpha_{\gamma\omega\nu} \Rightarrow$$

$$\underset{\alpha_{CM} = \alpha_{\gamma\omega\nu} \cdot R_\kappa}{\Rightarrow} \mathrm{T}_4 - \mathrm{T}_s = \frac{\mathrm{M}_\kappa}{2}\alpha_{CM} \quad (5)$$

$$(4) + (5) \rightarrow 2\mathrm{T}_4 - \mathrm{M}_\kappa g\eta\mu\phi = \frac{3}{2}\mathrm{M}_\kappa \alpha_{CM} \Rightarrow$$

$$\Rightarrow T4 = \frac{M_\kappa g\eta\mu\phi}{2} + \frac{3}{4}M_\kappa \alpha_{CM}\,(6)$$

$$T\rho o\chi\alpha\lambda\acute{\iota}\alpha : \Sigma\tau = I_\tau \alpha\gamma\omega\nu_\tau \qquad \Rightarrow T_3 R_T - T_4 R_T = \frac{M\tau R_T^{\ 2}}{2}\alpha_{\gamma\omega\nu,\tau}$$

$$T_3 - T_4 = \frac{M_T R_T}{2}\alpha_{\gamma\omega\nu,\tau}\,(7)$$

$$\Sigma\acute{\omega}\mu\alpha\Sigma : \Sigma F = m_\Sigma \alpha_\Sigma \Rightarrow m_\Sigma g - T_3 = m_\Sigma \alpha_\Sigma \Rightarrow$$

$$\Rightarrow T_3 = m_\Sigma g - m_\Sigma \alpha_\Sigma\,(8)$$

$$(7) \underset{(8)}{\overset{(6)}{\Rightarrow}} m_\Sigma g - \frac{m_\kappa g\eta\mu\varphi}{2} = m_\Sigma \alpha_\Sigma + \frac{3}{4}m_\kappa \alpha_{CM} + \frac{m_T R_T}{2} = \alpha_{\gamma\omega\nu,T}\,(9)$$

Όμως: $\Rightarrow \alpha_\Sigma = \alpha_{\gamma\omega\nu,T} \cdot R_T = 2\alpha_{CM}\,(10)$

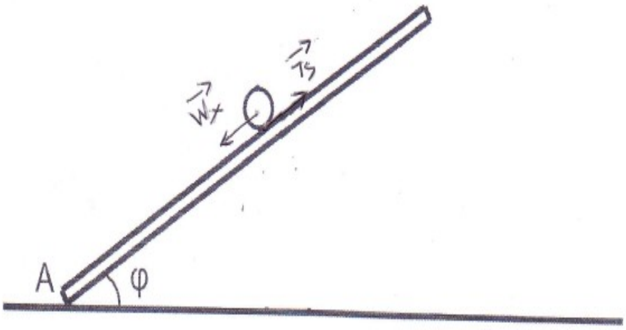
$$\underset{(10)}{\overset{(9)}{\Rightarrow}} \left(m_\Sigma - \frac{M_\kappa \cdot \eta\mu\varphi}{2}\right) \cdot g = \left(m_\Sigma + \frac{3}{8} \cdot M_\kappa + \frac{M_T}{2}\right) \cdot a_\Sigma$$

$$\underset{(10)}{\overset{(9)}{\Rightarrow}} \left(m_\Sigma - \frac{M_\kappa \cdot \eta\mu\varphi}{2}\right) \cdot g = \left(m_\Sigma + \frac{3}{8} \cdot M_\kappa + \frac{M_T}{2}\right) \cdot a_\Sigma$$

$$\Rightarrow \alpha_\Sigma = \frac{\left(2 - \frac{1}{2}\right)10}{2 + \frac{3}{4} + 1} = 4m/s^2$$

$$(10) \Rightarrow \alpha_{CM} - \frac{\alpha_\Sigma}{2} = 2m/s^2$$

Δ3.



Την $t_1 = 0,5s$ ο κύλινδρος έχει $u_{CM} = \alpha_{CM} \cdot t_1 = 1m/s$ και $\Delta x = \frac{1}{2}\alpha_{CM} t_1^2 = \frac{1}{2} \cdot 2 \cdot \frac{1}{4} = \frac{1}{4}m$

Αμέσως μετά

$$\Sigma\tau = I_\kappa \cdot \alpha'_{\gamma\omega\nu} \Rightarrow -T_s \cdot R_\kappa = \frac{M_K \cdot R_K^2}{2} \cdot a'_{\gamma\omega\nu} \Rightarrow -T_s = \frac{M_K}{2} \cdot a'_{CM} \quad (11)$$

$$\Sigma F = M_\kappa a'_{cm} \Rightarrow -M_\kappa \cdot g \cdot \eta\mu\varphi + T_S = M_\kappa \cdot a'_{CM} \quad (12)$$

$$(11)+(12) \Rightarrow -M_\kappa \cdot g\eta\mu\varphi = \frac{3}{2} M_\kappa \cdot \alpha'_{CM} \Rightarrow a'_{CM} = -\frac{2}{3} \cdot g\eta\mu\varphi \Rightarrow$$

$$\Rightarrow \alpha'_{CM} = -\frac{2}{3} \cdot 10 \cdot \frac{1}{2} \cdot m/s^2 = -\frac{10}{3} m/s^2$$

$$0 = u_{CM} + a'_{CM}\Delta t \Rightarrow \Delta t = -\frac{u_{CM}}{a'_{CM}} = \frac{1}{10/3} = 0,3s$$

Άρα στιγμιαία σταματά την. $t_2 = 0,8s$

Δ4.

$$\Delta x'_{CM} = u_{CM} \cdot \Delta t + \frac{1}{2}\alpha'_{CM}\Delta t^2 \Rightarrow \Delta x'_{CM} = 1 \cdot 0,3 \cdot - \frac{1}{2} \cdot \frac{10}{3} \cdot 0,3^2 \Rightarrow$$

$$\Rightarrow \Delta x'_{CM} = 0,15m$$

Άρα $\Delta x_{CM} = 0,4m$

Δ5. Στην ακραία θέση του ο κύλινδρος ασκεί στη ράβδο δύναμη $\mathrm{N}' = \mathrm{M}_{\kappa} \cdot g \cdot \sigma \upsilon \nu \varphi$

η οποία δημιουργεί ροπή $\mathrm{N}' \cdot d$ ως προς το Γ όπου d = 0,4- 0,2= 0,2m.

$$\tau'_{\mathrm{N}} = \mathrm{N}' \cdot d = \mathrm{M}_{\kappa} \cdot g \cdot \sigma \upsilon \nu \varphi \cdot d = 20 \sigma \upsilon \nu \varphi \cdot 0,2 = 4 \sigma \upsilon \nu \varphi \cdot Nm$$

Η ροπή του βάρους της ράβδου ως προς το Γ:

$$\tau_{W\rho} = M_{\rho} \cdot g \left(\mathrm{A\Gamma} - \frac{1}{2} \right) \sigma \upsilon \nu \varphi = 2 \cdot 10 \cdot 0,5 \cdot \sigma \upsilon \nu \varphi = 10 \sigma \upsilon \nu \varphi \cdot \mathrm{N}m$$

Αφού $\left| \tau_{W\rho} \right| > \left| \tau_{N} \right|$ άρα δεν ανατρέπεται.

ΕΠΙΜΕΛΕΙΑ
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